Algebra 2

Lesson 4-Snow: The Last word on Matrices

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Not covered in our Chapter 4 lessons is a set-up for matrices called an **augmented matrix**. When given a system of equations, instead of putting them into 3 matrices: coefficient, variable, and constant, and use the inverse matrix to solve for x-y-z, we can place the coefficients and constants into one matrix. R**emember** that the first columns of elements are the coefficients of our system of equations. To help us to remember that the last column is the coefficient column there is a vertical line drawn separating the last column.

Example: Place the system into augmented matrix form:

$$\begin{aligned} 2x + y + z &= 3 \\ x - 2y - z &= 0 \\ x + 2y + z &= 0 \end{aligned} \qquad \left[\begin{array}{ccc|c} 2 & 1 & 1 & 3 \\ 1 & -2 & -1 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right]$$

Ok, so now what? There are two matrix methods that may be used, first method is manual (don't groan). The second method, however, is readily done on the calculator. Both methods manipulate the matrices to get them into what is known as **row-echelon form** and **reduced row-echelon form**. Manually, the matrix is manipulated in a similar fashion as to what we do in elimination; however it takes generally takes a lot of steps. We add rows together in such a way so as to get zeros and ones for the numbers in the matrix. This is done by multiplying a scalar by a row, add or subtract the rows, and repeat the process until we get a matrix into one of the following forms. *Plan on doing this in college; for today this is a calculator exercise.*

Note: The calculator does all the calculations for you! 

row-echelon form: **reduced row-echelon form:**

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 12 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

What do you notice about these two matrices? The zeros? Yes. In the row-echelon form the matrix has now been transformed such that we have a solution for z and the x and y values may quickly be calculated:

$$\begin{aligned} x + y + 3z &= 12 \\ y - 2z &= -4 \\ z &= 3 \end{aligned}$$

The reduced row-echelon is even better! $\boldsymbol{x} + 0y + 0z = 12$ We have values for x, y, and z.

$0x + \boldsymbol{y} + 0z = -4$

$0x + 0y + \boldsymbol{z} = 3$ so our solution is $(12, -4, 3)$

How can this be done on a calculator? Actually quite straight forward:

1. Under **EDIT** key in the dimensions for the system of equations. If x, y, and z, then there will be 3 rows and 4 columns. $\mathbf{3 \times 4}$
2. Enter in the coefficients and constant values
3. **2nd QUIT** then **2nd x^{-1}** to get back into matrices. **ARROW>** to **MATH** and down to **A: ref** (**r**ow-**e**chelon **f**orm) or **B: rref** (**r**educed **r**ow-**e**chelon **f**orm).**ENTER**
4. **2nd x^{-1} [A] ENTER ENTER** and you have your matrix answer

Solve the following systems by getting them into reduced row-echelon form:

$$\begin{cases} -4x + 4y - 5z = 15 \\ -x - 2y - 4z = 3 \\ -2x - 3y - 4z = -3 \end{cases}$$

x=

y=

z=

$$\begin{cases} -2w - 2x + y = 8 \\ -w + 2x + y + 2z = -4 \\ 2w + x + 3y - z = -6 \\ -2w - 3x - y - 3z = 6 \end{cases}$$

$$\begin{cases} 2x - y - 5z = -12 \\ 4x - 5y + z = -2 \\ -5x - 2y - 5z = -5 \end{cases}$$

x=

y=

z=

$$\begin{cases} -3w - x - 3y = -27 \\ w + x + 3y + z = 17 \\ 2w - 2x + y - z = 9 \\ -2w + 3x - 3y + z = -16 \end{cases}$$