Algebra 2
Lesson 5-6: Complex Numbers
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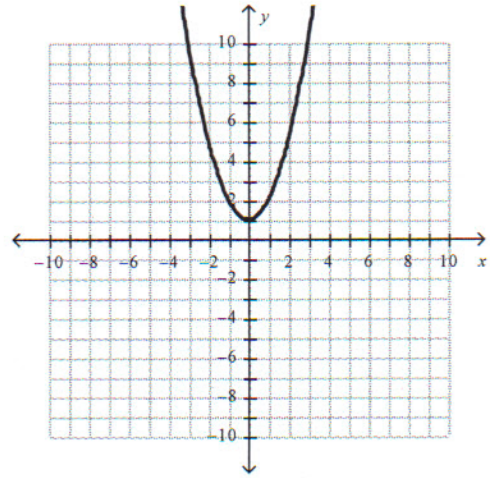
In order to solve equations and describe situations and the associated data, people invented numbers. Many of these number ideas were rejected or were met with resistance and were regarded as nonsense. In fact, the number 0 was not invented at the same time as the natural numbers. As a matter-of-fact, the Romans had no numeral for the value 0! There were probably skeptics who wondered why it necessary to have a number to represent nothing. Then of course negative numbers met resistance as well. How could someone possibly need to count – 6 oxen? The same was true for complex numbers. The catch is that complex numbers are no more imaginary than any other number created by mathematicians.

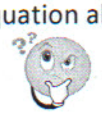


If you take a look at the graph of $y = x^2 + 1$, you will see that it has no x-intercepts.

$\sqrt{x^2} = \sqrt{-1}$ $\quad x = \sqrt{-1}$

Therefore, the equation $x^2 + 1 = 0$ has no real solutions.

Attempting to solve this equation algebraically, we get



$x^2 = -1, thus\ x = \sqrt{-1}$ up to this point we have regarded this as having no solution, well this is true in part. It has no *real solution*, but it does have an *imaginary solution* as defined by:

$\boldsymbol{i = \sqrt{-1}}$
$\boldsymbol{i^2 = -1}$ $\rightarrow$ $i^2 = (\sqrt{-1})^2$

$i^2 = -1$

Now, by inventing ***i***, the solutions to the equation $x^2 + 1 = 0$, $x = \pm\sqrt{-1} = \pm i$

Complex numbers may be defined by combining real numbers and the imaginary unit *I*, ***a*** *and* ***b*** are real numbers, including 0

$\boldsymbol{a + bi}$ — $\sqrt{-1}$ $\quad 0+2i \leftarrow 2i$

$\sqrt{-4}$ $\quad i \quad 2$

$\sqrt{-1\cdot 4} = \sqrt{-1}\,\sqrt{4}$

real **imaginary**

Can a real number be a complex number? Yes! Because a real number may be written as $a + 0i$. When b≠0, we get an imaginary number: **bi: examples: 3i, -2i, ½i,** and so on. .

$5+0i$

Example: Simplify: $\sqrt{-12} =$ $\sqrt{-1\cdot 6\cdot 2} = \sqrt{-1\cdot 2\cdot 3\cdot 2}$ $\sqrt{-1}\cdot\sqrt{2\cdot 2}\cdot\sqrt{3}$ $i\cdot 2\sqrt{3}$ $= 2i\sqrt{3}$	1. Using the product rule, separate real and imaginary parts (negative) 2. Now by definition, simplify (remembering, $\boldsymbol{i = \sqrt{-1}}$) 3. And rearrange
$\cancel{\sqrt{-4}}$ $\sqrt{-16}$ $i\sqrt{16}$ ± 4 $\pm 4i$	$\sqrt{-7}$ $\sqrt{-1\cdot 7}$ $\pm i\sqrt{7}$

Likewise we can simplify complex numbers into the form $a + bi$

Example: Write the complex number in the form $a + bi$ Real imaginary $\sqrt{-18} + 7$ $\sqrt{-1 \cdot 9 \cdot 2}$ $\pm 3i\sqrt{2}$ $\underline{\underline{7 \pm 3i\sqrt{2}}}$	1. simplify the radical expression 2. Now write in the form $a + bi$

Adding Complex numbers: Add like terms, that is, add the real parts together and add the imaginary parts

$(2 + 3i) + (5 - i)$ $2 + 5 + 3i - i$ $\underline{\underline{7 + 2i}}$	$(12 + 5i) - (2 + i)$ $12 - 2 + 5i + i$ $\underline{\underline{10 + 6i}}$

Multiplying Complex numbers: *FOIL! remember!* $i^2 = -1$ $\quad x^2 = x \cdot x$

$(8 + i)(2 + 7i)$ $i \cdot 7 \cdot i$ $7 \cdot i \cdot i$ $7i^2$ $7(-1)$ -7 $16 + 56i$ $+ 2i + 7i^2$ (-1) $16 + 58i - 7$ $\underline{\underline{9 + 58i}}$	$(5 - 2i)^2$ $(5 - 2i)(5 - 2i)$ $(-2i)(-2i)$ $4i^2$ $4(-1)$ $\boxed{-4}$ $25 - 10i$ $-4 - 10i$ $21 - 20i$

Additive inverse is the sum of two numbers the equals 0. $\quad X + 5 = 6$

$-3i$ additive inv. $+3i$ $+3i - 3i = 0$	$-3 - 2i$ $\boxed{+3 + 2i}$ $0 + 0i$

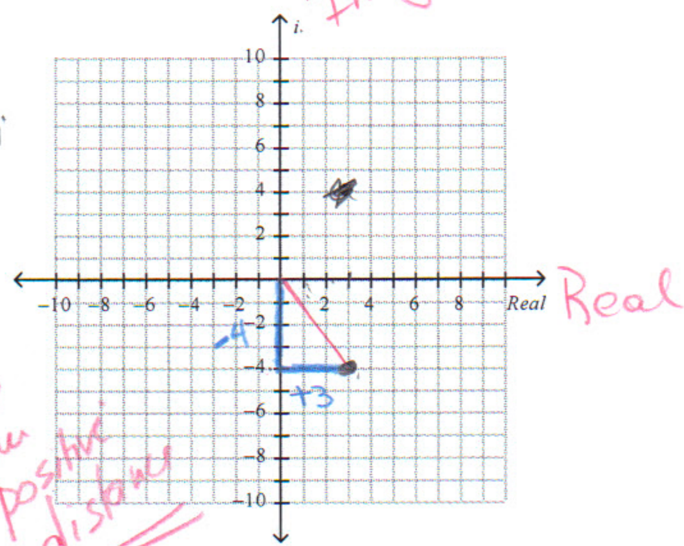
Graphing complex numbers

Graphing a complex number is very much like graphing points on an x-y coordinate system. Here **the horizontal axis is the real axis** and the **vertical axis is the imaginary axis:** When we graph $3 + 4i$, notice that a right triangle is made; the hypotenuse is the distance of our complex number from the origin. Distance from the origin is always positive. When did we look at distance from the origin? **Absolute values.**

The **absolute value of a complex number** is the distance from the origin on the complex number plane.
It is found by using the Pythagorean Theorem:

$$|\boldsymbol{a} + \boldsymbol{bi}| = \sqrt{\boldsymbol{a}^2 + \boldsymbol{b}^2}$$

3−4i



Example: Graph and find the absolute value of $|3 - 4i|$

1. plot 3 units to the right on the real axis and 4 units down on the imaginary axis

2. Distance to the origin is the hypotenuse of the triangle so:

$|3 - 4i| =$ $(3^2) + (-4^2) = hyp.^2$

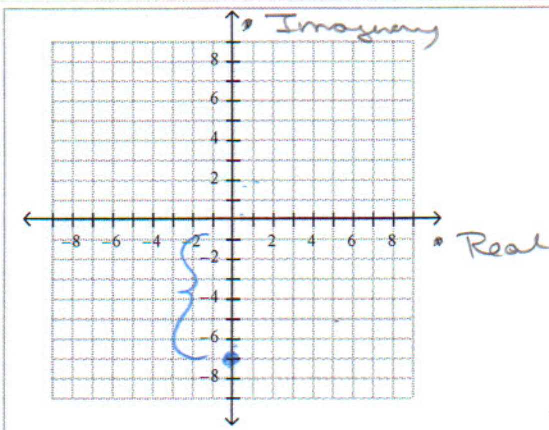
$9 + 16 = h^2$

$\sqrt{25} = \sqrt{h^2} \Rightarrow h = 5$

absolute value positive distance

Graph the complex number and find the absolute value of the complex number:

$|-7i|$ $= 7$

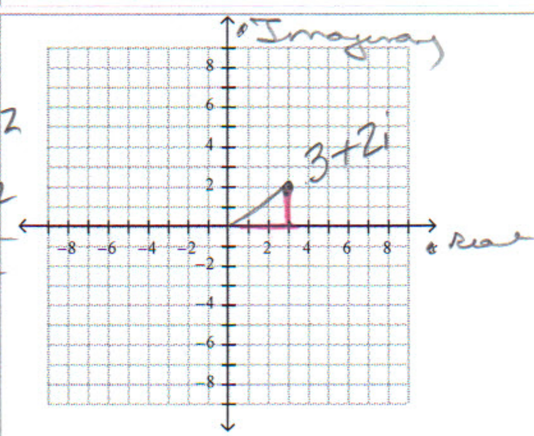


~~12i~~

$|3+2i| = 3^2 + 2^2 = h^2$

$= 9 + 4 = h^2$

$\boxed{|3+2i| = \sqrt{13}} = \sqrt{h^2}$



$|-5+6i| = h$

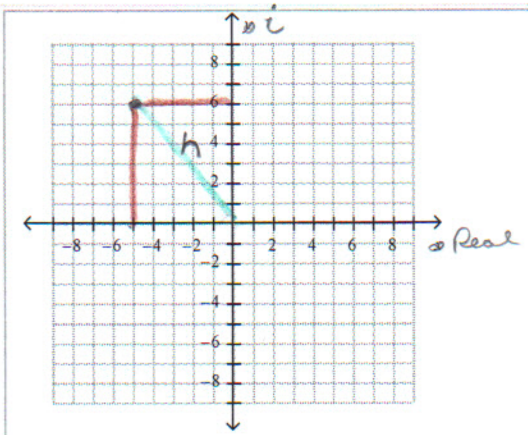
$(-5)^2 + 6^2 = h^2$

$25 + 36 = h^2$

$61 = h^2$

$\sqrt{61} = h$

$|-5+6i| = \sqrt{61}$



Solve by finding square roots

$\cancel{x^2 = -25}$

$x^2 + 25 = 0$

$\sqrt{x^2} = \sqrt{-25}$

$x = \pm 5i$

$3x^2 + 48 = 0$

-48

$\frac{3x^2}{3} = \frac{-48}{3}$

$\sqrt{x^2} = \sqrt{-16}$

$x = \pm 4i$