A quadratic function will have this form. Yes, “b” can equal 0 and “c” can equal 0. If “a” equals 0 then it is no longer a quadratic, but linear.

**Axis of symmetry** – a line that divides a parabola into two parts that are mirror images of each other. The axis of symmetry will be a vertical line with an equation in the form of \( x = \text{real number} \) and will be equal to the \( x \) value.

**Vertex** – is where the minimum or maximum value of the function and will occur at the value of the \( y \) point. This is the point where the direction of the parabola changes from decreasing to increasing or increasing to decreasing. \( \text{Vertex} (x, y) \text{ min or max } y = \pm \)

**Minimum or Maximum** – the value of \( y \) at the vertex

**Corresponding point** – points on a parabola that are the reflection of other points on the parabola.

e.g. on the above graph \( P \ (−2, −1) \) corresponds to \( P' \ ((0, −1)) \), plot \( P' \), What is \( Q' \)?

Identify the vertex, minimum or maximum, axis of symmetry and the domain and range for the graphs.

Identify the corresponding points for \( P \) and \( Q \)
If a calculator is allowed, you may find the minimum/maximum point of the parabola:

1. Using the Y= button enter the equation.
2. Hit GRAPH Note: the stat plots must be off for the graphing function to work.
3. Adjust the window of the view screen under ZOOM or WINDOW in order to view the vertex.
4. Hit $2^{nd}$ TRACE 3 minimum or 4 maximum. The view screen will ask for the left bound, arrow over so that the blinking star (asterisk) is on the left side of the vertex. ENTER You will be asked for the right bound, and again arrow over so that the asterisk is now on the right side of the vertex. ENTER ENTER and the view screen will identify the x and y coordinates for the vertex.

Example: Use the calculator to write a quadratic equation with the following points:

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>13</td>
<td>29</td>
</tr>
</tbody>
</table>

$$a = 3, \ b = -5, \ c = 1$$

$$\Rightarrow y = 3x^2 - 5x + 1$$

$$(1, -2), (2, -2), (3, -4)$$

$$a = -1 \quad b = 3 \quad c = -4$$

$$y = -x^2 + 3x - 4$$

*Here we use different methods to solve:*

**Method 1:**

1. A system of 3 equations and 3 unknowns may be solved with elimination or substitution.

**Method 2:**

1. Write as a matrix equation and solve.

**Method 3:**

1. Use the stat plot function on the calculator to plot the original given points
2. A quadratic regression (calculator) will yield the equation of best fit.
Method 1 and 2:

Given 3 ordered pairs, a quadratic equation may be found:

1. Substitute the values of x and y into the quadratic equation:
   \[ y = ax^2 + bx + c \]
2. With the 3 resultant equations you have a system of 3 linear equations and may be solved by methods learned in Chapter 3 and 4 or in the “Final Word on Chapter 4” lesson.
3. Using the augmented matrix form, key in the coefficients and constant into a 3x4 matrix on the calculator and find the reduced row-echelon form of the matrix, thus finding the solutions to the variables which are in fact the coefficients of the quadratic equation!

Method 3:

<table>
<thead>
<tr>
<th>LINEAR/QUADRATIC REGRESSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using STAT PLOT and finding a best fit line or curve:</td>
</tr>
<tr>
<td><strong>1.</strong> Given a set of data:</td>
</tr>
<tr>
<td><strong>2.</strong> Enter the Data into the calculator:</td>
</tr>
<tr>
<td><strong>3.</strong> Turn on STAT PLOT1</td>
</tr>
<tr>
<td><strong>4.</strong> Plot the data points</td>
</tr>
<tr>
<td><strong>5.</strong> Find the best fit and send the equation over to the y-plot select 4 for linear or 5 for quadratic regression.</td>
</tr>
<tr>
<td><strong>6.</strong> When Y= is opened you will see the equation has been placed for graphing, and a line will be drawn of best fit</td>
</tr>
</tbody>
</table>

**[STAT] [ENTER]**
Type in independent variable (x) data into L1, dependent data (y) data into L2, followed by 2nd [MODE] (quit)

**2nd [ Y=] [Enter] [Enter] 2nd [MODE]**
OR

**[Y=] ↑ Plot1 [Enter]**

**[STAT] [CALC] 5 [VARS] ▶ Y-VARS**

**[ENTER] [ENTER] [ENTER] [GRAPH]**

***IF YOU ONLY NEED AN EQUATION:***
**STAT ▶ CALC 4 ENTER** FOR A LINEAR
**STAT ▶ CALC 5 ENTER** FOR A QUADRATIC
Even with the limited knowledge we have from just completing one section, we can still come up with some information that describes a graph of a quadratic and choose an equation that represents the function modeled.

What is the vertex? \((-2, 7)\)

What is the y-intercept? \(y = -2\)

Leading coefficient, \(+/-\)?

- a. \(f(x) = -x^2 + 6x + 2\)
- b. \(f(x) = -x^2 + 6x - 2\)
- c. \(f(x) = -x^2 + 6x + 2\)
- d. \(f(x) = -x^2 + 6x - 2\)

What is the vertex? \((4, -7)\)

What is the y-intercept? \((0, 9)\)

Leading coefficient, \(+/-\)?

- a. \(f(x) = x^2 - 8x - 9\)
- b. \(f(x) = x^2 + 8x + 9\)
- c. \(f(x) = x^2 + 8x - 9\)
- d. \(f(x) = x^2 - 8x + 9\)

\(b\): \(-7 = 4^2 + 8(4) + 9\)

\(-7 = 16 + 32 + 9\)

\(-7 = 57\)

\(-7 = 4^2 - 8(4) + 9\)

\(-7 = 16 - 32 + 9\)

\(-7 = -7\)
PARABOLA STANDARD FORM:
\[ y = ax^2 + bx + c \]

1. When \( b = 0 \), the function is: \( y = ax^2 + c \). When graphed, the parabola will be symmetric around the y-axis. Therefore, the axis of symmetry is: \( x = 0 \), and the vertex of the graph is the y-intercept, \((0,c)\).
2. If \( a > 0 \) the parabola will open upward. \( a < 0 \), open downward.
3. The larger \( a \), the narrower the parabola. The smaller \( a \), the wider the parabola.
4. Setting \( x = 0 \) \( \rightarrow \) \( y = c \). \( c \) is the \( y \)-intercept!

To graph a quadratic equation in the form \( y = ax^2 + c \):
1. The vertex is at \((0,c)\). Note that this is also the y-intercept.
2. The sign of “\( a \)” tells us it opens up (+) or down (-).
3. Pick at least 3 points on one side of the vertex, solve for \( y \) and then find the corresponding points using symmetry to graph the other side.

Graph the function \( y = 2x^2 - 4 \)

Well, what if the equation is in standard form: \( y = ax^2 + bx + c \)?
1. The sign of the coefficient of \( a \) still tells us whether the parabola opens up (+) or down (-).
2. Axis of symmetry is now found from the coefficients of the equation, hence the axis is the line: \( x = \frac{-b}{2a} \)
3. The vertex of the parabola is at the point: \( x = \frac{-b}{2a} \), \( y = f\left(\frac{-b}{2a}\right) \);
   basically \( \text{when } x = \frac{-b}{2a} \), what is \( y \)?
4. Now, the parabola will be translated along the x-axis; however, the y-intercept is at \((0,c)\).
Example: Graph the function: \( y = 3x^2 + 6x - 4 \)

1. \( a = 3 \quad b = 6 \quad c = -4 \)

2. \( a > 0 \) or so opens \( \uparrow \)

3. \( y = \text{intercept} = c = -4 \)

4. Axis of symmetry: \( x = \frac{-b}{2a} = \frac{-6}{2(3)} = -1 \)
   \( -1 = x \) graph axis of symmetry.

5. Vertex: \( x = -1; \quad y = f \left( \frac{-5}{2(3)} \right) = f(-1) = -7 \)
   When \( x = -1 \), solve for \( y \)
   \[ y = 3(-1)^2 + 6(-1) - 4 = 3 - 6 - 4 = -7 \]
   \( \therefore (-1, -7), \text{vertex} \).

6. Select \( x \)-values adjacent to the axis of symmetry and find the corresponding \( y \)-value

\[
\begin{array}{c|cccccc}
 x & -4 & -3 & -2 & -1 & 0 & 1 & 2 \\
\hline
 y & 5 & -4 & -7 & -4 & 5
\end{array}
\]

7. Complete table with corresponding points reflected across the axis of symmetry.

8. The vertex is the location of the \underline{minimum/maximum}.
   What is this value? \( y = -7 \)

9. Domain: \( \text{All Real} \)
   Range: \( \left[ -7, \infty \right) \)

\[
\begin{align*}
3(1)^2 + 6(1) - 4 &= 7 - 18 - 4 \\
3(-3)^2 + 6(-3) - 4 &= 27 - 18 - 4 \\
&= 9 - 4
\end{align*}
\]

Calculator Minimums and Maximum:

1. Hit \( Y= \) type in the quadratic equation. \textbf{Remember}: must be in the "y=" form.

2. \textbf{GRAPH} if the parabola is off the view screen: \textbf{WINDOW} adjust the minimum and maximum values for \( x \) and \( y \). \textbf{GRAPH} and view the parabola.

3. \( ^{2}\text{ND TRACE} \) choose \( 3\)-minimum if the parabola is opening up or choose \( 4\)-maximum if the parabola is opening down. Question: \textbf{left bound}? Arrow over so that asterisk is flashing on the left side of the min or max \textbf{ENTER right bound}? Arrow over so that the asterisk is flashing on the right side of the min or max. \textbf{ENTER Guess}? \textbf{ENTER} the \( x \) and \( y \) values will be given at the bottom of the view screen.
Given the equation: \( y = -x^2 + 2x + 3 \)

1. \( a = -1 \), \( b = 2 \), \( c = 3 \)
2. \( a \) is \( \text{negative} \)
   so opens \( \text{down} \)
3. \( y = \text{intercept} = c = 3 \)
4. Calculate and graph Axis of Symmetry: \( x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1 \)
5. Calculate vertex
   \( y = -(1^2) + 2(1) + 3 \)
   \( = 4 \)
6. Make a table of values including y-intercept and vertex
   Choose 3 points on one side of the vertex.
   \( x \)
   \( 0 \)
   \( 1 \)
   \( 2 \)
   \( 3 \)
   \( y \)
   \( 3 \)
   \( 4 \)
   \( 3 \)
   \( 0 \)
7. Compete the table of values and graph.
8. State the minimum/maximum value.
9. Domain: \( \mathbb{R} \)
    Range: \( -\infty \leq y \leq 4 \)
    \( \left(-\infty, 4\right] \)
    \( -(3^2) + 2(3) + 3 \)
    \( -9 + 6 + 3 \)
Application:
The number of bacteria in a refrigerated food is given by \( n(t) = 20t^2 - 20t + 120 \), for \(-2 \leq t \leq 14\) and where \( t \) is the temperature of the food in Celsius. At what temperature will the number of bacteria be a minimum?

\[
\begin{align*}
\text{parabola opens up} & \quad \text{so} \quad \text{minimum} \\
\text{vertex (temp, bacteria)} & \quad \text{at} \quad \left( \frac{-b}{2a}, \frac{4ac-b^2}{4a} \right) = \left( \frac{-(-20)}{2(20)}, \frac{200-(-20)^2}{4(20)} \right) = \left( \frac{20}{40}, \frac{200-400}{80} \right) = \left( \frac{1}{2}, \frac{-200}{80} \right) = \left( \frac{1}{2}, -2.5 \right)
\end{align*}
\]

\( \Rightarrow \) so bacteria at minimum when \( t = \frac{1}{2}^\circ C \)

Nike Shoes estimates that its monthly profit \( P \) in hundreds of dollars can be modeled by the formula \( P = -2x^2 + 4x + 6 \), where \( x \) is the number of shoes produced per month in thousands. How many shoes should be produced per month to earn the maximum profit?

What is the maximum monthly profit?

\[
\begin{align*}
x &= -\frac{b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1 \\
\text{leading coefficient} &= -2 \\
\text{parabola opens down} & \quad \text{vertex at Maximum}
\end{align*}
\]

\[
\begin{align*}
P &= -2(1)^2 + 4(1) + 6 \\
 &= -2 + 4 + 6 \\
 &= 8 \quad \text{or} \quad \$800 \text{ profit}
\end{align*}
\]

A company's weekly revenue in dollars is given by \( R(x) = 2000x - 2x^2 \), where \( x \) is the number of items produced during a week. What amount of items will produce the maximum revenue?

\[
\begin{align*}
a &= -2 \quad \text{parabola opens down} \\
x &= -\frac{b}{2a} = -\frac{2000}{2(-2)} = -\frac{2000}{-4} = \frac{500}{1} = 500 \text{ number of items}
\end{align*}
\]

Note: max revenue at \( x = 500 \)

\[
\begin{align*}
R(x) &= 2000(500) - 2(500^2) \\
&= \$500,000 \text{ revenue}
\end{align*}
\]