## Algebra 2

## Lesson 4-7: Inverse Matrices and Systems

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Let's pick up with the question left unanswered from yesterday's lesson: what good is an inverse matrix? You can use the inverse of a matrix to solve a system of equations. This process is in fact quite similar to solving a general equation like: $5 x=20$. Multiply each side by $1 / 5$ (the inverse of 5 ) in order to isolate/solve for $x$. Wooo! We said the same thing yesterday! Today we are going to take the equations with $x, y$ and $z$, convert them into matrices, and solve.

Before we can solve a system of equations with matrices, we must convert the system to the matrix equation form: $A X=B$, where A is the coefficient matrix, X is the variable matrix, and B is the constant matrix.

Example: Represent the system of equations as a matrix equation and identify each part: Hold your horses! Before we begin let's identify the parts of a system of equations written in standard form!

SYSTEM:

$$
\left\{\begin{array}{c}
1 x+2 y=5 \\
3 x+5 y=14
\end{array}\right.
$$

Identify the coefficients, variables and constants:

Coefficients: 1, 2 and 3,5 Variables: $x$ and $y$ Constants: 5 and 14 Make sure to put these guys into their respective matrices!

## Matrix Equation:

$\left[\begin{array}{ll}1 & 2 \\ 3 & 5\end{array}\right] \quad\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{c}5 \\ 14\end{array}\right]$

| Coefficient | Variable | Constant |
| :---: | :---: | :---: |
| Matrix A | Matrix X | Matrix C |
| $\left[\begin{array}{ll}1 & 2 \\ 3 & 5\end{array}\right]$ | $\left[\begin{array}{l}x \\ y\end{array}\right]$ | $\left[\begin{array}{c}5 \\ 14\end{array}\right]$ |

notice: when you multiply the row $X$ column you get your equation back, that is why we put the variable $x$ in row 1 and $y$ in row $2 x+2 y=5$

## Steps for solving a system of equations:

1. Write the system as a matrix equation. Caution! Place the inverse in the correct location!
2. Find $A^{-1}$ (first find $\operatorname{det} A$ )
3. Multiply $A^{-1} B$
4. Solve for the variable matrix.

Example: Solve the system of equations by using matrices.

| $\left\{\begin{array}{c}x+2 y=5 \\ 3 x+5 y=14\end{array}\right.$ | 1. matrix equation (with inverse <br> included): <br> $A^{-1}\left[\begin{array}{ll}1 & 2 \\ 3 & 5\end{array}\right] \cdot\left[\begin{array}{l}x \\ y\end{array}\right]=A^{-1}\left[\begin{array}{c}5 \\ 14\end{array}\right]$$\quad$2a. <br> $\operatorname{det} \mathrm{A}=\left\|\begin{array}{ll}1 & 2 \\ 3 & 5\end{array}\right\|=$ <br> $5-6=-1$ |
| :---: | :---: |
| 2b. find inverse: $A^{-1}=\frac{1}{-1}\left[\begin{array}{cc} 5 & -2 \\ -3 & 1 \end{array}\right]=\left[\begin{array}{cc} -5 & 2 \\ 3 & -1 \end{array}\right]$ | 3. multiply through by inverse: $\begin{aligned} & {\left[\begin{array}{cc} -5 & 2 \\ 3 & -1 \end{array}\right]\left[\begin{array}{ll} 1 & 2 \\ 3 & 5 \end{array}\right]\left[\begin{array}{c} x \\ y=1 \end{array}\right]=\left[\begin{array}{cc} -5 & 2 \\ 3 & -1 \end{array}\right]\left[\begin{array}{c} 5 \\ 14 \end{array}\right]} \\ & {\left[\begin{array}{cc} -5 & 2 \\ 3 & -1 \end{array}\right] \cdot\left[\begin{array}{c} 5 \\ 14 \end{array}\right]=\left[\begin{array}{c} -25+28 \\ 15-14 \end{array}\right]=\left[\begin{array}{l} 4 \\ 1 \end{array}\right]} \end{aligned}$ <br> 4. solve <br> Answer $\quad X=\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}4 \\ 1\end{array}\right]$ the solutin to the system: $(4,1)$ |

Can we solve systems of equations in 3 variables? 4 variables? Yes, with the assistance of a calculator, we will!

Example: $\left\{\begin{array}{c}2 x+y+3 z=1 \\ 5 x+y-2 z=8 \\ x-y-9 z=5\end{array}\right.$

1. Write the system as a matrix equation
2. Enter the coefficient matrix as matrix $A$ and the constant matrix as matrix $B$ on the calculator.
3. Solve for $A^{-1}$ on the calculator
4. Enter $[A]^{-1} B$ into the calculator to solve for $(x, y, z)$

THINK:
The solution to a system of equations $(x, y)$ is a point where the two lines intersect. It is a unique solution as there are infinite solutions to each linear equation but only one solution where the two particular lines intersect. Likewise we have intersecting planes when faced with a system of 3 equations. In Ch. 3 we found that we could have situations where the lines were either the same line or parallel. For the planes we found they could intersect along the same line in space or not intersect at all. We had methods to determine this (our true or false equation that came out of the system of equations). Here we also have a method that will indicate no unique solution. The method focuses on the determinant. If the determinant equals $\mathbf{0}$ then there is no inverse; this indicates that the system has no unique solution (could be that the system is dependent or inconsistent).

PRACTICE: Solve each system of equations

| 1. $\left\{\begin{array}{l}x+5 y=-4 \\ x+6 y=-5\end{array}\right.$ | 3. $\left\{\begin{array}{c} 9 y+2 z=18 \\ 3 x+2 y+z=5 \\ x-y=-1 \end{array}\right.$ |
| :---: | :---: |
| 2. $\left\{\begin{array}{c}x+2 y=5 \\ 2 x+4 y=8\end{array}\right.$ | 4. $\left\{\begin{array}{c}-2 w+x+y=-2 \\ -w+2 x-y+z=-4 \\ -2 x+3 x 3 y+2 z=2 \\ w+x+2 y+z=6\end{array}\right.$ |

