## Algebra 2

## Lesson 3-6: Systems with Three Variables

## Mrs. Snow, Instructor

In Sec. 3.2 we studied systems made up of 2 equations. We discovered that when we solved both equations for a point ( $x, y$ ), that this point is not only the a solution to both equations, but also the point where the two lines intersect. Thus, it is a solution to the system of equations. What if we are given a system of 3 equations containing 3 variables, $x, y$, and $z$ ? Note that this system could not be graphed on the Cartesian plane; it may be graphed in a 3dimension system. At this time we will not concern ourselves with graphing the system to determine a solution; rather we will look at the algebraic methods studied in 3.2 and apply them to 3 equations.

In elimination you will combine 2 of the equations such that you eliminate a variable. Next you combine two other equations so to eliminate the same variable. Now you have 2 equations and 2 unknowns, solve just as you did in section 3.2 , then find the $3^{\text {rd }}$ variable.

## Example:

Solve the system of equations using the elimination method.

1. $x+y+z=6$
2. $2 x-y+3 z=9$
3. $-x+2 y+2 z=9$

Step 1 Add eq. 1 and 2 to eliminate y Step 2: Multiply 2 times eq 2 then add eq 2 and 3 to eliminate $y$.
Step 3: Take the results of Step 1 and 2, two equations with two unknowns and solve for the variables.
Step 4: Substitute the solutions for the two variables into an equation to solve for the $3^{\text {rd }}$ variable. .

Step 1: combine equ. 1 \& 2

1. $x+y+z=6$
2. $2 x-y+3 z=9$

$$
3 x+4 z=15
$$

Step 2: combine equ 2 \& 3
2. $2(2 x-y+3 z=9)$

$$
4 x-2 y+6 z=18
$$

3. $-x+2 y+2 z=9$

$$
3 x+8 z=27
$$

Step 3

$$
\begin{gathered}
3 x+4 z=15 \\
-(3 x+8 z=27) \\
\hline-4 z=-12 \\
z=\mathbf{3} \\
\boldsymbol{x}=\mathbf{1}
\end{gathered}
$$

Step 4 subbing $x$ and $z$ into:

$$
\begin{gathered}
x+y+z=6 \\
1+y+3=6 \\
y=2
\end{gathered}
$$

ANS: $(1,2,3)$
check:
$1+2+3=6 \sqrt{V}$
$2(1)-2+3(3)=9 \sqrt{ }$
$-1+2(2)+2(3)=9 \sqrt{ }$

The key is to analyze the equations to see which may be combined to eliminate a variable with little or no alteration.

In substitution, select an equation that can be easily solved in terms of one of the variables, for example solve $x$ in terms of $y$ and $z$. Next substitute the expression into the other remaining equations for the variable that you solved.

## Example:

Solve the system of equations using the substitution method.

$$
\begin{gathered}
x+y+z=-8 \\
x-y-z=6 \\
2 x-3 y+2 z=-1
\end{gathered}
$$

Step 1 Solve one equation in terms of the other variables.
Step 2 Substitute this term into both remaining equations for the variable. Step 3 Write the 2 equations as a system and solve as outlined in 3.2 Step 4 Once the two variable are solved, the first variable may then be calculated.

$$
\begin{aligned}
& \text { Step } 1 \\
& x=y+z+6 \\
& \text { Step } 2 x+y+z=-8 \\
& 2 y+2 z=-14 \\
& 2 x-3 y+2 z=-1 \\
& 2(y+z+6)-3 y+2 z=-1 \\
& 2 y+2 z+12-3 y+2 z=-1 \\
& -y+4 z=-13
\end{aligned}
$$

## Step 3

multiply second by 2 and add to first:

$$
\begin{gathered}
2 y+2 z=-14 \\
-2 y+8 z=-26 \\
\hline 10 z=-40 \\
z=-4 \\
y=-3
\end{gathered}
$$

Step 4

$$
\begin{gathered}
x-3-4=-8 \\
x=-
\end{gathered}
$$

Ans (-1, -3, -4)
check:
$-1-3-4=-8$
$-1+3+4=6$
$2(-1)-3(-3)+2(-4)=-1$

