We have all seen the little model cars that children have, you probably even had several of your own. These model cars replicate real cars, there are even car kits that are advertised as exact scale models, that is they look just like the real thing only smaller. Well, in the math world we also have many models. Data collected from physical and natural phenomena can be quantified and placed into a mathematical model so to duplicate a process or predict an outcome of a future event.

In this section we will focus on linear models. These are models that use a linear function to describe the data. For example a car travelling at a constant speed which is the rate a.k.a. slope, if we plot time (x) and distance (y), based on the slope of the line we can predict the distance travelled in a given amount of time.

Example: A person is riding a bicycle along a straight highway. The accompanying graph shows the rider’s distance y in miles from an interstate highway after x hours.

Modeling with a linear function takes on a basic form. To model a quantity that is changing at a constant rate in a linear function f, we use:

\[ f(x) = (\text{constant rate of change})x + (\text{initial amount}) \]

Look familiar? This is our slope-intercept form. The constant rate of change is the ____________ and the initial amount is the _______________.

You can use the equation of slope, \( m = \frac{y_2-y_1}{x_2-x_1} \) when you are given two data points.
Example: After 1 hour burning a candle is 6 in. tall, 3 hours later it is 5 in. tall. Use \((3, 5)\) for \((t_1, h_1)\) and \((1, 6)\) for \((t_2, h_2)\) in the equation for slope. Calculate the slope and plug the slope and one of the points into the point-slope equation, solve for \(y\) and will have the equation of a line that models the height of the candle.

Example: A 100-gallon tank is initially full of water and is being drained at a rate of 5 gallons per minute.
a) write a formula for a linear function \(f\), that models the number of gallons of water in the tank after \(x\) minutes. follow the steps below by answering these questions and draw a picture of what is happening:

Step 1: What is happening (driving, object falling, ice melting, babies born, water draining, etc.)?  
Ans:

Step 2: What then is the constant rate of change?  
Ans: __________

Step 3: What is the initial amount? (initial distance, temperature, bacteria count, gallons in the tank, etc.)  
Ans: 100- gallons

Step 4: Build the equation:  
\[ f(x) = -5x + 100 \]  
Caution!! This one is tricky as the rate of change is negative: are we adding or taking away water? We are taking away/draining water therefore, the rate of change for a decreasing amount is negative!

b) How much water is in the tank after 4 minutes?  
Plug 4 minutes into the above equation and solve for \(f(x)\):  
\[ f(x) = -5(4) + 100 = -20 + 100 = 80 \text{ gallons} \]
c) Graph \(f\). Identify the \(x\)- and \(y\)-intercepts

Not all data are linear. When plotted the data may appear to be close to linear, in which case a linear function may be used to approximate the data. Simply plot the data on the graph, this known as a scatter plot. Next, draw what appears to be a best fit line, that is, some points above the line, some below, and some points on the line. There are computer and calculator programs that will plot the data in a scatter plot and calculate the best fit line as well. Remember the more linear the data the more accurate your linear model approximation will be.
Example: The table shows the distance traveled in miles by car using $x$ gallons of gasoline.

a) Make a scatter plot of the data. Can a linear function be used to model this data?

b) Find values for $a$ and $b$ so that $f(x) = ax + b$ models the distance traveled on $x$ gallons and graph the plot of that line.

c) Interpret the slope of the graph $f(x)$.

<table>
<thead>
<tr>
<th>$x$ (gallons)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$ (miles)</td>
<td>84</td>
<td>169</td>
<td>255</td>
<td>338</td>
</tr>
</tbody>
</table>

**Calculator:** To calculate a linear equation to fit the data:

- **STAT** **EDIT**<E> enter $x$ in L1 and $y$ in L2
- **2nd** **Y=** turn on stat plot 1, hit **ZOOM 9** (zoomstat) <E>

**STAT** **CALC** 4 **VARS** **Y-VARS** 1<E> <E>

You should get an equation $y = ax + b$ and the values for $a$ and $b$. This equation with the VARS commands has also been sent over to the Y= plot and when you hit graph, the line of best fit will be graphed in too.