A job at a manufacturing plant pays $10.00. This table shows the relationship between the hours worked and dollars earned. Here we see that dollars earned is equal to the number of hours worked multiplied by 10.

<table>
<thead>
<tr>
<th>Hours worked</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dollars earned</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

This concept is known as **direct variation** so, as $x$ increases $y$ increases at a constant rate. As $x$ decreases $y$ also decreases, at a constant rate. This can be expressed in a linear equation. The general form is: $y=kx$ and if you think about it, it makes sense that $k \neq 0$.

**Examples:** Graph the following equations:

- $y=3x$
- $y=\frac{1}{2}x$
- $y=-4x$
- $y=2x$
- $y=-3x$
- $y=-\frac{1}{2}x$

What do you notice about the constant $k$? Is it equal to something previously studied??

Previously, we studied tables of $x$ and $f(x)$ data to determine if the data represented a line. We can also determine whether $y$ varies directly with $x$.

**Note:** When we determine if an equation demonstrates $y$ varying directly with $x$, it **must have the form:** $y=kx$. Therefore while the equation $y=mx+b$ is linear, $y$ does not vary directly with $x$.

Given: $y=kx$ solve for $k$

$\frac{y}{x}=k$ so, if we look at the ratio $y/x$ for each point $(x,f(x))$ and all ratios are equal, the given equation $y=kx$ will in fact vary directly.

**Examples:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

$$\frac{3}{1}=\frac{6}{2}=\frac{12}{4}=3$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>-2</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>
1. Find the constant of proportionality \( k \) and the undetermined value so that \( y \) is directly proportional to \( x \).
   a)
   \[
   \begin{array}{c|c|c|c|c}
   x & 3 & 5 & 6 & 8 \\
   y & 7.5 & 12.5 & 15 & ? \\
   \end{array}
   \]
   
   \[
   \begin{array}{c|c|c|c|c}
   x & 1.2 & 4.3 & 5.7 & ? \\
   f(x) & 3.96 & 14.19 & 18.18 & 23.43 \\
   \end{array}
   \]

2. For each function determine whether \( y \) varies directly with \( x \). Graph and label equations.
   a)
   \[
   \begin{array}{c|c|c|c|c}
   x & 3 & 2 & 6 & -5 \\
   y & 2 & 3/4 & 4 & -10/3 \\
   \end{array}
   \]
   b)
   \[
   \begin{array}{c|c|c|c}
   x & 1 & 2 & 3 & -3.8 \\
   f(x) & .5 & 2 & 4.5 & -7.22 \\
   \end{array}
   \]

4. The cost of tuition is directly proportional to the number of credits taken.
   a) Write an equation stating this relation.
   b) If 11 credits costs $720.00, what is the constant of proportionality?
   c) What is the cost of taking 16 credits?

4. The maximum load that a horizontal beam can carry is directly proportional to its width. If a beam 1.5 inches wide can support a load of 250 pounds, find the load that a beam of the same type can support if its width is 3.5 inches.

5. The weight of an object on Earth is directly proportional to the weight of an object on Mars. If a 25-lb. object on Earth weighs 10 lbs. on Mars, how much would a 195 lb. astronaut weigh on Mars?

3. Does \( y \) vary directly with \( x \)? If so what is \( k \)?
   a) \( y=5x \)
   b) \( 3y=4x \)
   c) \( -\frac{3}{4}x=2y \)
   d) \( y=6x-5 \)
   e) \( y+1=4x \)

6. \( y \) varies directly with \( x \).
   a) If \( y=4 \) when \( x=-2 \), find \( x \) when \( y=8 \)
   b) If \( y=6 \) when \( x=2 \), find \( x \) when \( y=9 \)
   c) If \( y=3 \) when \( x=4 \), find \( y \) when \( x=2.4 \)
   d) If \( y=-10 \) when \( x=2 \), find \( y \) when \( x=-6/5 \)