Algebra II

Lesson 10-6: Translating Conic Sections

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We have seen with our conics sections that they are not limited to being centered about the vertex, but may be translated about the x – y planes; the center is at (h, k). The table below summarizes the equations for the conics sections both centered at the origin and how a translation will alter the standard form:

Conics Section	Center/vertex at $(0,0)$	Center/Vertex at (h, k)	Foci
Circle (10.3)	Center $(0,0)$ $x^2 + y^2 = r^2$	Center (h, k) $(x-h)^2 + (y-k)^2 = r^2$	NA
Ellipse (10.4) $\boldsymbol{a^2 - b^2 = c^2}$	Center $(0,0)$ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	Center (h, k) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$	$(h \pm c, k)$ $(h, k \pm c)$
Hyperbola (10.5) $\boldsymbol{a^2 + b^2 = c^2}$	Center $(0,0)$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	Center (h, k) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	$(h \pm c, k)$ $(h, k \pm c)$
Parabola (10.2) $\boldsymbol{\lvert a \rvert = \frac{1}{4c}}$	Vertex $(0,0)$ $y = ax^2$ $x = ay^2$	Vertex (h, k) $y = a(x-h)^2 + k$ $x = a(y-k)^2 + h$	$(h, k \pm c)$ $(h \pm c, k)$

Let's do a review of our translation problems:

Write an equation of an ellipse with a center of (1, -4), horizontal major axis of length 10 and minor axis of length 4	We need to understand some terminology first. 1. Length of the major axis: that is from vertex to vertex or **2a,** we need just **a** 2. Identify the values of (h,k) 3. Which is the major axis? 4. Plug into the standard formula.

<table>
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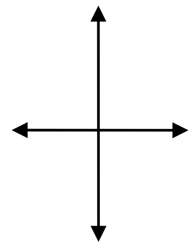
<td>Write the equation of a hyperbola with vertices (2,-1) and (2,7) and foci (2,10) and (2,-4).</td>
<td>1. Sketch the points so you know the orientation of the hyperbola. Vertices show a vertical orientation.

2. Where is the midpoint of a line through the vertices? Use the midpoint formula to find y.

3. What is the foci? With a and c what is b?</td>
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<td>Identify the conic section with the following equation: $x^2 + y^2 - 12x + 4y = 8$</td>
<td>1. Group like terms together (x and y) and locate the constant on the right side of the equation

2. Complete the square ½ of the linear term coefficient squared, add to both sides of the equation
3. If we divide through by the constant to get a 1 for the standard form of an ellipse we will see that the axes are equal, hence we have a circle with a radius of $\sqrt{48}$</td>
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Try: $4x^2 + 9y^2 + 16x - 54y = -61$

After we group like terms together and set about to complete the square, we need to understand how to handle a coefficient in front of the quadratic term.

1. Factor the leading coefficient out, if at all possible. S**quare ½ the linear coefficient and then multiply that product with outside factor.** For x, ½ of 4 squared is 4 multiply by the outside 4. What is added to the left, add to the right…

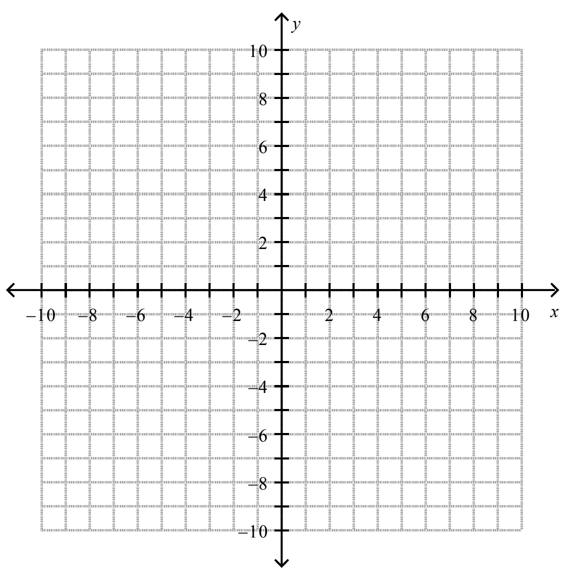
2. follow through and simplify

3. What is the conic?

In Algebra I and in the 1st semester of algebra II we calculated the intersection of two lines; i.e. solving a system of equations. Well, system of equations may involve conic equations; they are not reserved for linear equations.

Solve the system of equations by graphing: 

$$\begin{cases} \dfrac{x^2}{16} + \dfrac{y^2}{49} = 1 \\ 7x + 4y = 28 \end{cases}$$



$$\begin{cases} x^2 + y^2 = 36 \\ -x + y = 6 \end{cases}$$