## Algebra II

## Lesson 10-5: Hyperbolas

## Mrs. Snow, Instructor

In this section, we will look at the hyperbola. A hyperbola is a set of points P in a plane such that the absolute value of the difference between the distances from $P$ to two fixed points $F_{1}$ and $F_{2}$ is a constant k :
$\left|P F_{1}-P F_{2}\right|=k$, where $k<F_{1} F_{2}$ The points $\mathbf{F}$ are the foci of the hyperbola. There are two basic forms of a hyperbola.

Here are examples of each. The hyperbolas are centered at the point $(h, k)$; here we see this is the origin. .

Horizontal opens up along the x-axis:

$$
\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1 \text { or } \frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1
$$



Vertical opens up along the $y$-axis:

$$
\frac{y^{2}}{a^{2}}-\frac{x^{2}}{b^{2}}=1 \text { or } \frac{(y-k)^{2}}{a^{2}}-\frac{(x-h)^{2}}{b^{2}}=1
$$



Hyperbolas consist of two concave shaped pieces that open either left and right or up and down. Like parabolas each of the branches have a vertex. Note: they are not parabolas.

Hyperbola - is centered about the point ( $h, k$ )
Vertex - The point on each branch closest to the center. The vertices are the end points of the transverse axis and are a fixed distance a from the center. Note: In the standard form of a hyperbola, the denominator of the leading term is $\boldsymbol{a}^{2}$.

Transverse axis -A line segment going from one vertex, through the center, and ending at the other vertex. The foci are on this line.
$>$ The end points are the vertices of the hyperbola.
$>$ The midpoint, $(h, k)$, of this line segment is the center of a hyperbola.
Foci - Located "inside" each branch, and each focus is located some fixed distance $\mathbf{c}$ from the center.
$>$ Foci found by using the relationship: $\boldsymbol{c}^{2}=\boldsymbol{a}^{2}+\boldsymbol{b}^{2}$
$>$ This means that $\boldsymbol{a}<\boldsymbol{c}$ for hyperbolas.)
$>$ The values of $a$ and $c$ will vary from one hyperbola to another, but they will be fixed values for any given hyperbola.

Asymptotes - There are also two crossed lines on each graph. These lines are the asymptotes. They make a large $X$ (one with a positive slope and the other with a negative slope).
$>$ The graph of the hyperbola gets closer and closer to the asymptotes.
$>$ While the asymptotes are not officially part of the graph of the hyperbola, they are necessary for accurate graphing of a hyperbola. AND!!Rrequired for graphing problems!
$>$ The slope of the asymptotes is always the square root of the numbers under the $y$ term divided by the square root of the number under the $x$ term $\left(\frac{\text { rise }}{\text { run }}\right)$.
$>$ The point where the two asymptotes cross is called the center of the hyperbola.

Find the vertices, foci, and asymptotes of the hyperbolas. Graph.
$\frac{x^{2}}{36}-\frac{y^{2}}{4}=1$

Write an equation of a hyperbola:
foci $( \pm 5,0)$; vertices $( \pm 3,0)$

## 10.6

There are two standard forms of the hyperbola, one for each type shown above. Here is a table giving each form as well as the information we can get from each one. Note, I have set this up for a hyperbola whether centered about the origin or not.

| Opens | Opens left and right <br> along the x-axis | Opens up and down <br> along the $\mathbf{y}$-axis |
| :---: | :---: | :---: |
| Form: | $\frac{(x-h)^{2}}{a^{2}}-\frac{(y-k)^{2}}{b^{2}}=1$ | $\frac{(y-k)^{2}}{a^{2}}-\frac{(x-h)^{2}}{b^{2}}=1$ |
| Center: | $(h, k)$ | $(h, k)$ |
| Vertices | $(h+a, k) a n d(h-a, k)$ | $(h, k+a) a n d(h, k-a)$ |
| Slope of <br> Asymptotes | $y-k= \pm \frac{b}{a}(x-h)$ | $y-k= \pm \frac{a}{b}(x-h)$ |
| Equation of <br> Asymptotes | $(h+c, k),(h-c, k))$ | $(h, k+c),(h, k-c)$ |
| Foci <br> $a^{2}+b^{2}=c^{2}$ |  |  |

$>\mathbf{a}$ is first in the equation. It goes with the direction of the axis

- Opens horizontal - first term is $x$ and $a$
- Opens vertical - first term is $y$ and $a$

The equations of the asymptotes come from the point-slope form of the line and the fact that we know that the asymptotes will go through the center of the hyperbola. OR use the values of $a$ and $b$ to find and graph the vertices and to draw a central rectangle that is used as a guide to draw the asymptotes.

Find the vertices, foci, and asymptotes of the hyperbolas. Graph.

## Sketch the graph of the following hyperbolas:

$$
\frac{(x-3)^{2}}{25}-\frac{(y+1)^{2}}{49}=1
$$

1. Opens:
2. Center coordinates from the $h$ and $k$
3. $a=\quad b=$
4. Vertices:
5. The slope of the asymptotes is always the square root of the numbers under the $y$ term divided by the square root of the number under the $x$ term.
6. The foci $a^{2}+b^{2}=c^{2}$.


| Write an equation of a hyperbola with vertices $(2,2)$ and $(-4,2)$; Foci $(6,2)$ and $(-8,2)$ | Ok, so here is a graph. As a minimum always draw a sketch so you can get your bearings (are you doing the problem correctly? |
| :---: | :---: |

Writing in Standard Form

$$
x^{2}-4 y^{2}-2 x-8 y=7
$$

