Algebra II Lesson 10-5: Hyperbolas Mrs. Snow, Instructor

In this section, we will look at the hyperbola. A hyperbola is a set of points P in a plane such that the absolute value of the difference between the distances from P to two fixed points F_1 and F_2 is a constant k:

 $|PF_1 - PF_2| = k$, where $k < F_1F_2$ The points **F** are the foci of the hyperbola. There are two basic forms of a hyperbola.

Here are examples of each. The hyperbolas are centered at the point (h, k); here we see this is the origin.



Hyperbolas consist of two concave shaped pieces that open either left and right or up and down. Like parabolas each of the branches have a vertex. **Note:** *they are not parabolas.*

Hyperbola - is centered about the point (h, k)

Vertex - The point on each branch closest to the center. The vertices are the end points of the transverse axis and are a fixed distance <u>a from the center</u>. Note: In the standard form of a hyperbola, the denominator of the leading term is a^2 .

Transverse axis – A line segment going from one vertex, through the center, and ending at the other vertex. The foci are on this line.

- > The end points are the vertices of the hyperbola.
- > The midpoint, (h, k), of this line segment is the center of a hyperbola.

Foci – Located "inside" each branch, and each focus is located some fixed distance <u>c from the</u> <u>center</u>.

- > Foci found by using the relationship: $c^2 = a^2 + b^2$
- This means that *a* < *c* for hyperbolas.)
- The values of a and c will vary from one hyperbola to another, but they will be fixed values for any given hyperbola.

Asymptotes – There are also two crossed lines on each graph. These lines are the **asymptotes**. They make a large X (one with a positive slope and the other with a negative slope).

- > The graph of the hyperbola gets closer and closer to the asymptotes.
- While the asymptotes are not officially part of the graph of the hyperbola, <u>they are</u> <u>necessary for accurate graphing of a hyperbola</u>. AND!!Rrequired for graphing problems!
- > The slope of the asymptotes is always the square root of the numbers under the y term divided by the square root of the number under the x term $\left(\frac{rise}{run}\right)$.
- > The point where the two asymptotes cross is called the center of the hyperbola.

Find the vertices, foci, and asymptotes of the hyperbolas. Graph.



foci $(\pm 5, 0)$; vertices $(\pm 3, 0)$

10.6

There are two **standard forms** of the hyperbola, one for each type shown above. Here is a table giving each form as well as the information we can get from each one. Note, I have set this up for a hyperbola whether centered about the origin or not.

Opens	Opens left and right along the x-axis	Opens up and down along the y-axis
Form:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center:	(h,k)	(h,k)
Vertices	(h+a,k)and $(h-a,k)$	(h, k + a) and $(h, k - a)$
Slope of Asymptotes	$\pm \frac{b}{a}$	$\pm \frac{a}{b}$
Equation of Asymptotes	$y - k = \pm \frac{b}{a}(x - h)$	$y - k = \pm \frac{a}{b}(x - h)$
Foci $a^2 + b^2 = c^2$	(h + c, k), (h - c, k))	(h, k + c), (h, k - c)

> **a** is first in the equation. It goes with the direction of the axis

- Opens horizontal first term is *x* and *a*
- Opens vertical first term is *y* and a

The equations of the asymptotes come from the point-slope form of the line and the fact that we know that the asymptotes will go through the center of the hyperbola. **OR** use the values of a and b to find and graph the vertices and to draw a central rectangle that is used as a guide to draw the asymptotes.

Find the vertices, foci, and asymptotes of the hyperbolas. Graph.

Sketch the graph of the following hyperbolas:

$$\frac{(x-3)^2}{25} - \frac{(y+1)^2}{49} = 1$$

- 1. Opens:
- 2. Center coordinates from the *h* and *k*

3.
$$a = b =$$

- 4. Vertices:
- 5. The slope of the asymptotes is always the square root of the numbers under the *y* term divided by the square root of the number under the *x* term.

6. The foci
$$a^2 + b^2 = c^2$$
.





Writing in Standard Form

