Algebra II
Lesson 10-5: Hyperbolas
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In this section, we will look at the hyperbola. A hyperbola is a set of points P in a plane such that the absolute value of the difference between the distances from P to two fixed points F_1 and F_2 is a constant k:
$|PF_1 - PF_2| = k, where\ k < F_1F_2$ The points **F are the foci** of the hyperbola. There are two basic forms of a hyperbola.

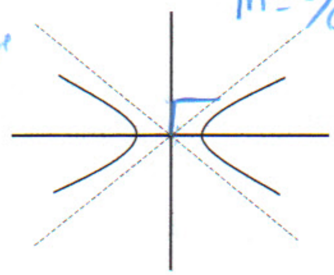
Here are examples of each. The hyperbolas are centered at the point (h, k); here we see this is the origin. .

Horizontal opens up along the x-axis:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad or \quad \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

X positive term orientation

$m = b/a$ $\frac{y}{x}$

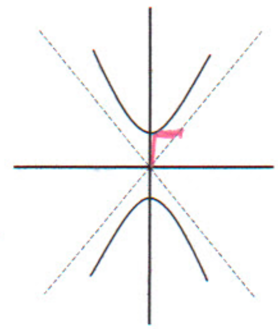


Vertical opens up along the y-axis:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad or \quad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$m = \frac{a}{b}$

y + term orientation



Hyperbolas consist of two concave shaped pieces that open either left and right or up and down. Like parabolas each of the branches have a vertex. **Note:** *they are not parabolas.*

Hyperbola - is centered about the point (h, k)

Vertex - The point on each branch closest to the center. The vertices are the end points of the transverse axis and are a fixed distance <u>a from the center</u>. Note: In the standard form of a hyperbola, the denominator of the leading term is $\boldsymbol{a^2}$.

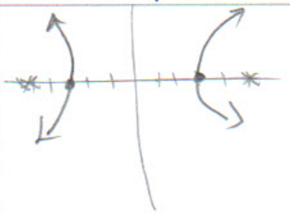
Transverse axis –A line segment going from one vertex, through the center, and ending at the other vertex. The foci are on this line. Major axis - hyperbola opens on this axis

- The end points are the vertices of the hyperbola.
- The midpoint, (h, k), of this line segment is the center of a hyperbola.

Foci – Located "inside" each branch, and each focus is located some fixed distance <u>**c** from the center</u>.

- Foci found by using the relationship: $\boldsymbol{c^2 = a^2 + b^2}$
- This means that ***a*** **<** ***c*** for hyperbolas.)
- The values of *a* and *c* will vary from one hyperbola to another, but they will be fixed values for any given hyperbola.

Write an equation of a hyperbola:



$foci\ (\pm5, 0)\ ;\ vertices\ (\pm3, 0)$

c

Horizontal

"a"

$a = 3 \quad c = 5$

$a^2 = 9 \quad c^2 = 25$

$c^2 = a^2 + b^2$

$25 = 9 + b^2$

$16 = b^2 \Rightarrow b = \pm 4$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Hyperbola orientation is 1st term

10.6

There are two **standard forms** of the hyperbola, one for each type shown above. Here is a table giving each form as well as the information we can get from each one. Note, I have set this up for a hyperbola whether centered about the origin or not.

Opens	**Opens left and right along the x-axis** x 1st ⇒ horizontal	**Opens up and down along the y-axis** y 1st ⇒ vertical
Form:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center:	(h, k)	(h, k)
Vertices	$(h + a, k) and\ (h - a, k)$	$(h, k + a) and\ (h, k - a)$
Slope of Asymptotes	$\pm\frac{b}{a}$	$\pm\frac{a}{b}$
Equation of Asymptotes	$y - k = \pm\frac{b}{a}(x - h)$	$y - k = \pm\frac{a}{b}(x - h)$
Foci $a^2 + b^2 = c^2$	$(h + c, k)\ , (h - c, k))$	$(h\ , k + c)\ , (h\ , k - c)$

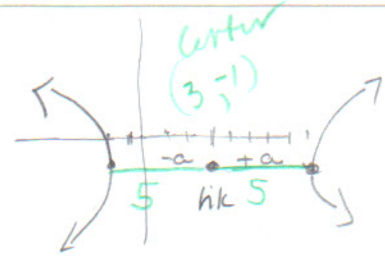
- **a** is first in the equation. It goes with the direction of the axis
  - Opens horizontal – first term is $x\ and\ a$
  - Opens vertical – first term is $y\ and\ a$

The equations of the asymptotes come from the point-slope form of the line and the fact that we know that the asymptotes will go through the center of the hyperbola. **OR** use the values of $a\ and\ b$ to find and graph the vertices and to draw a central rectangle that is used as a guide to draw the asymptotes.

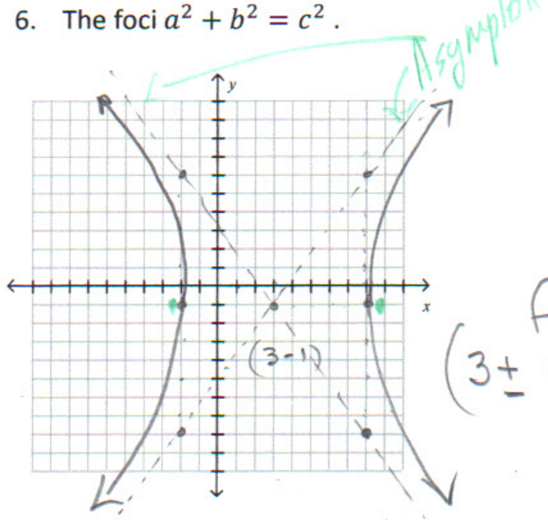
Find the vertices, foci, and asymptotes of the hyperbolas. Graph.

Sketch the graph of the following hyperbolas:

$$\frac{(x-3)^2}{25} - \frac{(y+1)^2}{49} = 1$$



1. Opens: Horizontal (x 1st)
2. Center coordinates from the *h and k* (3, -1)
3. $a =$ 5 $\quad b =$ 7
4. Vertices: (3+5, -1) (3-5, -1) => (8, -1) & (-2, -1)
5. The slope of the asymptotes is always the square root of the numbers under the *y* term divided by the square root of the number under the *x* term. $\frac{7}{5}$
6. The foci $a^2 + b^2 = c^2$.



$25 + 49 = c^2 = 74$

$c = \pm\sqrt{74}$

on graph
between 8 & 9

foci
$(3 \pm \sqrt{74}, -1)$

Write an equation of a hyperbola with
$vertices\ (2,2)\ and\ (-4,2);\ Foci\ (6,2) and\ (-8,2\ )$

$$\frac{(x+1)^2}{9} - \frac{(y-2)^2}{40} = 1$$

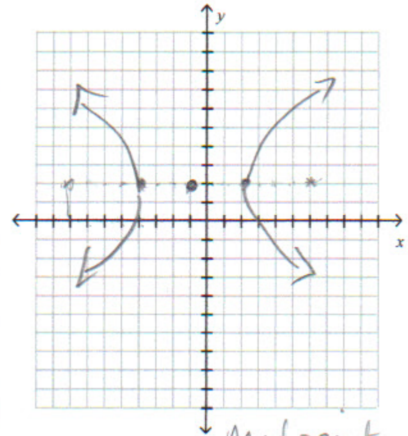
$a = 3 \quad c = 7$

$6+1$

$c^2 = a^2 + b^2 \quad 49 = 9 + b^2$

$40 = b^2$

Ok, so here is a graph. As a minimum always draw a sketch so you can get your bearings (are you doing the problem correctly?



Midpoint

$\frac{-4+2}{2} = \frac{-2}{2} = -1$

Center $(-1, 2)$
$(h\ k)$

Writing in Standard Form

$$x^2 - 4y^2 - 2x - 8y = 7$$

$$x^2 - 2x + 1 \quad -4y^2 - 8y - 4 = 7 + 1 - 4$$

$$-4(y - 2y + 1)$$

$$\frac{(x-1)^2}{4} \quad \frac{-4(y-1)^2}{4} = \frac{4}{4}$$

$$\frac{(x-1)^2}{4} - \frac{(y-1)^2}{1} = 1$$

center at $(1, 1)$

$c^2 = 4 + 1 = 5$
$c = \pm\sqrt{5}$

$(1+\sqrt{5}, 1)$ foci